

A Dynamic Model of a V-Shaped Free-Piston Engine

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Received November 10, 2010

Abstract—A model of one of the classes of a free-piston Stirling engine, namely a two-piston engine with V-shaped positioning of cylinders, is considered. The differential equations describing the dynamics of the engine as a self-oscillating system are derived taking sluggishness of the moving engine elements (pistons) and redistribution of the actuating medium volumes in the cylinders into account.

DOI: 10.3103/S1052618811020075

Internal combustion engines operated on liquid and gaseous fuel (piston and gas turbine ones) are currently used most widely in different branches of industry. However, during several recent decades scientific and practical interest, both in Russia and abroad, is being paid to the machines (engines included) operated based on another principle, namely external heating using a Stirling cycle [1–4]. Such engines have a number of advantages as compared to internal combustion engines, including circulation of the working medium in a closed volume, high efficiency, a low level of noise (the audible range included), and a possibility to use any heat source.

The Stirling engine is a heat engine operating by a closed thermodynamic cycle where all thermodynamic processes of compression and expansion take place at different temperature levels and a flow of the working medium is controlled by varying its volume. From a design point of view, the Stirling engine consists of expansion and compression cylinders, a regenerator, and a cooler. It should be noted that the majority of Russian and foreign papers devoted to the analysis of the Stirling engines that have been published by now give only a thermodynamic analysis of their operation cycle. The investigation of displacement of their moving elements (pistons) has been performed, as a rule, based on the kinetostatic analysis, i.e., without taking sluggishness of their moving elements into account.

The authors of [5, 6] took dynamics of moving elements into account and, based on this, the differential equations of their motion at different sections of a closed cycle were derived for a free-piston two-cylinder engine with coaxial cylinders. The calculations performed based on such model make it possible to introduce substantial quantitative and qualitative corrections into the piston motion laws.

It follows from the thermodynamic calculations, without dynamics of piston displacement taken into account, that during the second (isochronous compression) and the fourth (isochronous expansion) cycle sections the pistons in both cylinders move with identical velocities preserving a constant distance between one another. The dynamic calculation performed in [5, 6] shows that such motion is impossible. This confirms the importance of taking the dynamics of moving elements into account when calculating work processes in Stirling engines.

In the present paper, a dynamic model is calculated for a free-piston V-shaped Stirling engine of simple operation (Fig. 1): (1, 3) working pistons; (2) a generator; h is the piston thickness, $z = x$ is the length of the inner piston surfaces, l is the length of branch pipes, and d is the branch pipe diameter.

As in [5, 6], the following assumptions were made when developing the model: hydraulic, mechanical, and heat losses are absent; all processes are reversible; regeneration is ideal; a steady state mode of engine an operation is considered. As distinct from a free-piston engine considered in [5, 6], the dead zone for such engine type turns out to be non-zero (Fig. 1).

An ideal operating cycle of the engine consists of four sections (moves): curve 1–2 correspond to isothermal compressions; curve 2–3, to isochronous compression; curve 3–4, to isothermal expansion; and curve 4–1, to isochronous expansion.

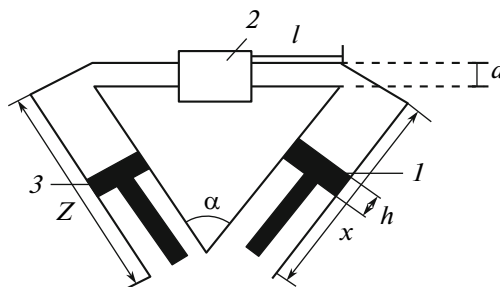


Fig. 1.

The X symbol marks the distance from the extreme low position of the piston in the right cylinder (compression cavity) to its extreme top position (Fig. 1), where x is the current displacement of this piston. The Z symbol marks the distance from the extreme low position of the piston in the left cylinder (expansion cavity) to its top position, where z of the current displacement of the left piston.

The dynamics equations for the four cycle sections are written as follows. In the first cycle section (isothermal expansion), the first piston creates a force under the action of the external pressure p_0 that exceeds that formed by the working medium pressure. The left piston remains immobile. In this case, the current volume of the working medium (with the branch pipe volume taken into account) is

$$V_r(x) = S\left(X - x - \frac{h}{2}\right) + \pi d^2 l + \pi \frac{d^3}{2} \tan \frac{\alpha}{2}. \quad (1)$$

In the function x , the current value of $V_r(x)$ (1) linearly decreases from $V_{r \max}(h/2) = SX + \pi d^2 l + \pi \frac{d^3}{2} \tan \alpha$, corresponding to the extreme low position of the piston to $V_{\min}\left(X - \frac{h}{2}\right) = \pi d^2 l + \pi \frac{d^3}{2} \tan \frac{\alpha}{2}$. In this case, the current value of the pressure $p_r(x)$ in the compression cavity increases. If we consider the working medium to be an ideal gas, the pressure $p_r(x)$ and the current value of the volume $V_r(x)$ (1) are interrelated with the Mendelev–Clapeyron equation

$$p_r(x) V_r(x) = G_0 R T_{\min}, \quad (2)$$

where G_0 is the total mass of the working medium, $R = R\mu/M$ is the universal gas constant, μ is the specific gas constant, M is the molar mass of the working medium, and $T_{\min}$ is the working medium temperature in the right cylinder.

Based on Eq. (2), let us determine the expression for the current value of the pressure in the compression cavity

$$p_r(x) = \frac{G_0 R T_{\min}}{V_r(x)}. \quad (3)$$

Thus, two forces act on the piston in the right cylinder: the external force $p_0 S$ from below, and the force $p_r(x) S$ (3) from above (outwards the right cylinder). Since these forces are oppositely directed (along the cylinder axis), the resultant force acting on the piston is $F_{tr} = p_0 S - p_r(x) S$. Thereby, the differential equation for the piston motion has the form

$$m\ddot{x} = p_0 S - \frac{G_0 R T_{\min}}{V_r(x)} S. \quad (4)$$

It is necessary to integrate Eq. (4) with the entry conditions: $x(0) = X - \frac{h}{2}$, $\dot{x}(0) = 0$. The duration of the first cycle section is determined from the condition of the piston halt $\dot{x}(t_1) = 1$. Based on this, the piston coordinate at the halt moment is determined, $x_{10} = x(t_1)$.

At the second cycle section, both pistons move. The left piston starts moving down (from the regenerator), while the right piston moves up. The current value of the working medium volume in the expansion cavity is determined with the equation

$$V_l(z) = S\left(Z - z - \frac{h}{2}\right) + \pi d^2 l_r + \pi \frac{d^3}{2} \tan \frac{\alpha}{2}. \quad (5)$$

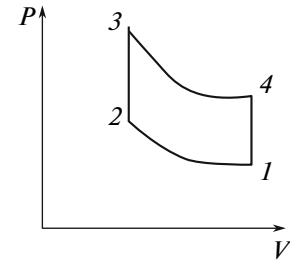


Fig. 2.

It is seen from the cavity volume that expansion decreases. In this case, the working medium extruded by the right piston passes via the regenerator to the left cavity where, as the pistons move, its temperature increases according to the dependence obtained in [5]

$$T_l(z) = T_{\min} + \frac{\alpha S}{G_0 C_v} (z_1 - z), \quad (6)$$

where $z_1 \leq z - h/2$ is the coordinate of the extreme top position that was occupied by the left piston before the beginning of the second cycle section; C_v is the specific heat of the working medium, and α is the coefficient of proportionality between the amount of the heat and the volume of the working medium passing via the regenerator in the process of its flow from the compression cavity (the right cylinder) to the expansion cavity (the left cylinder). According to Eq. (6), when the left cylinder goes down, the temperature of the working medium in the expansion cavity increases.

The values of $V_l(z)$ (5) of the pressure $p_l(z)$ and $T_l(z)$ (6) in the expansion cavity are interrelated by the Mendelev–Clapeyron equation

$$p_l(z) V_l(z) = G_l(z) R T_l(z), \quad (7)$$

where $G_l(z) = G_0 - G_r(x)$ is the current mass of the working medium in the expansion cavity and $G_r(x)$ is the current value of the working medium volume in the right cylinder (the compression cavity).

In this cycle section, G_0 in the Mendelev–Clapeyron equation for the right cylinder has to be replaced by $G_r(x)$ and, as the result, it takes on the form

$$p_r(x) V_r(x) = G_r(x) R T_{\min}. \quad (8)$$

From Eqs. (7) and (8), let us determine the pressures in the left and right cavities

$$p_l(z) = \frac{G_l(z) R T_l(z)}{V_l(z)}, \quad p_r(x) = \frac{G_r(x) R T_{\min}}{V_r(x)}. \quad (9)$$

According to [7], the velocity of the working medium flow from the right cavity to the left one via the regenerator is related to the pressure difference in the cavities by the following equation

$$p_l(z) - p_r(x) = \gamma \frac{dG_l(z)}{dt}, \quad (10)$$

where γ is the coefficient of the regenerator resistance.

The motion equations for the left and the right pistons have the form

$$m\ddot{z} = [p_l(z) - p_0]S, \quad m\ddot{x} = [p_0(z) - p_r(x)]S. \quad (11)$$

Equations (10) and (11) form a closed set of nonlinear differential equations with respect to three functions: the current values of the coordinates of the right and left pistons $x(t)$, $z(t)$, and the current value of the working medium mass in the left piston $G_l(x, z, t)$.

The entry conditions for this set of equations (the moment of the beginning of the second cycle t_1 corresponding to the end of the first cycle) are specified as follows: $G_l(x_1, z_1, t_1) = 0$ and $z(t_1) = z_1$, $x(t_1) = x_1$.

The moment of the end of the second cycle section is determined from the condition $G_l(x_2, z_2, t_2) = G_l(t_2) = G_0$; i.e., all working medium has overflowed to the left cylinder. The corresponding piston coordinates $z_2 = z(t_2)$, $x_2 = x(t_2)$ determine the extreme low position of the left piston and the extreme top position of the right piston. In this case, the temperature in the expansion cavity reaches its maximum determined, according to Eq. (6), with the formula

$$T_{\max} = T_l(z_2) = T_{\min} + \frac{\alpha S}{G_0 C_v} (z_1 - z_2). \quad (12)$$

The temperature in the cavity of the left cylinder at the third cycle section remains constant and is equal to the maximal value $T_{\max}$ (12). In this case, the right piston remains immobile ($x_2 = x(t_2)$), while the left one continues going down owing to an increase in the working medium volume in the process of its heating at the constant temperature $T_{\max}$.

The volume of the working medium $V_l(z)$ determined with Eq. (5) and the pressure $p_l(z)$ are interrelated by the Mendelev–Clapeyron equation $p_l(z) V_l(z) = G_0 R T_{\max}$. From here, let us determine the pressure in the function z : $p_l(z) = G_0 R T_{\max} / V_l(z)$.

The motion equation for the left piston is

$$m\ddot{z} = [p_1(z) - p_0]S. \quad (13)$$

Equation (13) has to be integrated with the entry conditions $z(t_2) = z_2$ and $\dot{z}(t_2) = \dot{z}_2$, where $\dot{z}_2$ is the velocity of $\dot{z}$ determined from the solution of set of equations (10) and (11) of the previous cycle section calculated at the moment of its termination t_2 .

The end of the third cycle corresponds to a halt of the left piston $\dot{z}(t_3) = 0$. From this condition, the position of the left piston which it occupies in the end of the third cycle section is determined ($z_3 = z(t_3)$).

The fourth section is calculated in the manner analogous to that for the second section. The difference is as follows: the left piston is now going up, while the right one goes down, and the working medium flows via the regenerator from the left cylinder to the right one.

The set of differential equations describing the dynamics of this section is written as

$$m\ddot{x} = [p_r(x) - p_0]S, \quad m\ddot{z} = [p_0 - p_l(z)]S, \quad p_l(z) - p_r(x) = \gamma \frac{dG_1(z)}{dt}. \quad (14)$$

It is necessary to integrate the set of differential equations (14) with the entry conditions $x(t_3) = x_2$, $\dot{x}(t_3) = 0$, $z(t_3) = z_3$, $\dot{z}(t_3) = 0$, and $G_1(x_2, z_3) = G_0$.

The moment of the fourth section termination or the period of the entire cycle T is equivalently determined from the conditions $x(T) = x_1$, $z(T) = z_1$, and $G_1(x_1, z_1, T) = 0$.

Thus, the cycle dynamics in the first and the third section is described by differential equations (4) and (13), respectively; that in the second and fourth sections is described by differential equation sets (10), (11) and (14), respectively. Though differential equations (4) and (13) are integrated in quadratures, the corresponding analytical equations are rather complex. As for the differential equation sets (10), (11) and (14), they are not integrated in quadratures.

Thus, it is necessary to use numerical methods for the analysis of the cycle dynamics.

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